Math 220 - Calculus f. Business and Management

Solutions for Worksheet 1 - Introduction to Functions

Simple Business Problems

Exercise 1: *A company is planning to make dishes. The initial cost to begin manufacturing is* $\$950,000$. *The raw material and labor for each plate is* $\$4.00$. *The raw material and labor for each bowl is* $\$2.00$. *What is the cost to make* $100,000$ *plates and* $50,000$ *bowls?*

Solution to #1:

Fixed cost $C_F = \$950,000$, *Cost per plate* $C_P = \$4.00$, *cost per bowl* $C_B = \$2.00$.

$$\text{Total cost } C = C_F + 100,000C_P + 50,000C_B = \$950,000 + \$400,000 + \$100,000 = \boxed{\$1,450,000}$$

Exercise 2: *The company in problem 1 will sell each plate for* $\$8.00$ *and each bowl for* $\$7.00$. *How much revenue will the company receive by selling all the plates and bowls? Will the company make a profit at this volume of plates and bowls? If so, how much profit will it make? If not, what are its losses?*

Solution to #2:

Price per plate: $P_P = \$8.00$, *Price per bowl* $P_B = \$7.00$,

$$\text{Revenue } R = 100,000P_P + 50,000P_B = \$800,000 + \$350,000 = \boxed{\$1,150,000}$$

Cost exceeds revenues by $\$1,450,000 - \$1,150,000 = \$300,000$. *The company makes a* $\boxed{\text{loss of } \$300,000.00}$

Exercise 3:

If the company were to only make plates, how many plates would it need to break even?

Solution to #3:

Denote the quantity of plates sold by q *and revenue as a function of the quantity sold by* $R(q)$ *and the cost to sell a quantity of* q *plates by* $C(q)$. *Then* $R(q) = \$8.00q$ *and* $C(q) = \$950,000 + \$4.00q$. *The break-even quantity* q_0 *is the one for which* $C(q0) = R(q0)$, *i.e.,* $950,000 + 4q_0 = 8q_0$. *We solve for* q_0: $950,000 = 4q_0$, *i.e.,* $\boxed{q_0 = 237,500}$ *plates is the quantity at which the company breaks even.*

Geometric Cost Problems

Exercise 4:

Carpet costs $\$20$ *per square meter. What is the cost to carpet*

a. *a rectangular room that is* 3 *meters by* 4 *meters?*
b. *A circular room that has a radius of* 2 *meters?*
c. *A space that is a right triangle with legs of length* 4 *meters and* 3 *meters?*

Solution to #4a (rectangular room):

Area is $3m \times 4m = 12m^2$, *hence cost is* $12m^2 \times \$20.00$ *per* $m^2 = \boxed{\$240.00}$

Solution to #4b (circular room):

Area is $r^2\pi = 4\pi m^2$, *hence cost is* $4\pi m^2 \times \$20/m^2 = \boxed{\$80.00\pi} \approx \$80 \times 3.14 = \$251.20$

Solution to #4c (triangular room):

A rectangular triangle has area $(1/2)\times$ *leg*$_1 \times$ *leg*$_2$ *because if you consider one leg as the base line then the other leg will be the height of the triangle. Hence the area is* $(4 \cdot 3)/2 \;=\; 6m^2$ *and it follows that the cost is*
$6m^2 \times \$20.00$ *per* $m^2 \;=\; \boxed{\$120.00}$.

Exercise 5: *Fencing to surround each of the shapes in the previous problem costs $15.00 per meter. What is the cost to surround each of the shapes?*

Solution to #5a (rectangular room):

Perimeter is $2 \times 3m \;+\; 2 \times 4m \;=\; 14m$*, hence cost is* $14m \times \$15.00$ *per meter* $= \boxed{\$210.00}$

Solution to #5b (circular room):

Circumference is $2r\pi = 4\pi$ *meters, hence*
cost is $4\pi m \times \$15/m \;=\; \$60.00 \times \pi \;\approx\; \$60 \times 3.14 = \boxed{\$188.40}$

Solution to #5c (triangular room):

The hypothenuse has length $\sqrt{4^2 + 3^2}m \;=\; 5$ *meters (use Pythagoras).*
Hence the circumference is $3 + 4 + 5 = 12$ *and the cost is* $12m \times \$15.00/m \;=\; \boxed{\$180.00}$

Exercise 6:

A rectangular prism has sides of 30 *cm,* 25 *cm and* 40 *cm. Material to cover the prism costs* $\$1.50$ *per square cm (*cm^2*). How much will it cost to cover all six sides (surface areas) of the prism?*

Solution to #6:

$$\begin{aligned}
\textit{Surface area} &= 2(30 \times 25cm^2 \;+\; 30 \times 40cm^2 \;+\; 40 \times 25cm^2) \\
&= 2(750 + 1,200 + 1000)cm^2 \;=\; 5,900cm^2, \\
\textit{Cost} &= 5,900cm^2 \times \$1.50/cm^2 \;=\; \boxed{\$8,850.00}
\end{aligned}$$

Exercise 7:
a. *Use the information from the previous problem to find the cost to cover a cylinder that has a radius of* 2 *cm and a height of* 6 *cm.*
b. *What would it cost to cover a sphere with a radius of* 3 *cm?*

Solution to #7a:

Surface area of a cylinder of radius $r = 2cm$ *and height* $h = 6cm$ *is*
$2r^2\pi\; cm^2 \;+\; 2h\pi r\; cm^2 \;=\; (8\pi + 24\pi)\; cm^2 \;=\; 32\pi cm^2$.
Hence the cost is $32\pi cm^2 \times \$1.50/cm^2 \;=\; \boxed{\$48.00\pi} \;\approx\; \$150.72$.

Solution to #7b:

Surface area of the sphere is $4\pi r^2 \;=\; 36\pi cm^2$*. Hence the cost is* $36\pi cm^2 \times \$1.50/cm^2 \;=\; \boxed{\$54\pi} \;\approx\; \$169.56$.

Exercise 8: *Suppose you have containers in the shapes described in the previous two problems. Liquid to fill the containers costs* $\$0.10$ *per cubic centimeter (*cm^3*). How much will it cost to fill each of the containers?*

Solution to 8a (prism):
Volume of the prism is $30cm \times 25cm \times 40cm \;=\; 30,000cm^3$*. Hence the cost is* $30,000cm^3 \times \$0.10/cm^3 \;=$ $\boxed{\$3,000.00}$

Solution to 8b (cylinder):
Volume of the cylinder is $\pi r^2 h = 24\pi cm^3$. *Hence the cost is* $24\pi cm^3 \times \$0.10/cm^3 = \boxed{\$2.4\pi} \approx \$7.536 \approx \$7.54$.

Solution to c (sphere):
Volume of the sphere is $4/3\pi r^3 = 36\pi cm^3$. *Hence the cost is* $36\pi cm^3 \times \$0.10/cm^3 = \boxed{\$3.6\pi} \approx \$11.304 \approx \$11.30$.