Math 220 - Calculus f. Business and Management - Worksheet 2

Solutions for Worksheet 2 - Constructing Functions

Simple Linear Functions

Exercise 1: *A train is $50km$ (kilometers) from its home terminal and traveling away in a straight line at 80 kph. Write the equation that shows its distance from home as a function of how much time has passed. Use this equation to find out how far it will be from home after 3 hours? How long will it take for the train to be $320km$ from home?*

Solution to #1:

The train starts $50km$ from home, so our function will have a constant term of 50. The train travels $80kph$, so our function will have an $80t$ term. Notice t is given in hours, since the units are kph. So the distance function is $D(t) = 50 + 80t$.

To find where the train will be after 3 hours, plug 3 into the function found above. You get $D(3) = 50+80(3) = 290km$ away.

To find the time it takes for the train to travel $320km$, set your distance function equal to 320, and then solve for t. You get $50 + 80t = 320$. To solve, subtract 50 from both sides and then divide by 80. You get $t = \frac{270}{80} = 3.375$ hours.

Exercise 2: *The price of gas is \$3.89 per gallon. If it increases at a rate of \$0.09 per week, express the price as a function of time in weeks.*
How long before gas reaches \$5.00 per gallon? (Hint: at $t = 0, p = \$3.89$)

Solution to #2:

There is a constant term of 3.89, since that is the price we are starting at. The price increases \$0.09 per week, so we have another term of $0.09t$. So the price function is $P(t) = 3.89 + 0.09t$.

To see how long it takes the price to become \$5.00, set the price function equal to 5.00. So you have $3.89+0.09t = 5.00$. Subtract 3.89 from both sides and divide by 0.09 to obtain $t = \frac{1.11}{0.09} = 12\frac{1}{3}$ weeks.

Geometric Problems

Exercise 3:

A farmer wants to fence in a rectangular enclosure. The enclosure will be twice as long as it is wide. The material for the shorter sides costs \$3.00 per ft (foot). The material for the longer sides costs $\$5.00/ft$. Write an equation for the cost of the fence as a function of the length of the shorter side. How much will it cost to enclose an area that is 20 feet wide? If the farmer has \$1300, how big an area can he enclose?

Solution to #3:

The rectangle is twice as long as it is wide. If the shorter sides have length w feet, then the longer sides have length $2w$ feet. Each foot of the shorter sides costs \$3.00 and each foot of the longer sides costs \$5.00. So the cost function is $C(w) = 3(w) + 3(w) + 5(2w) + 5(2w) = 26w$.

To find the cost of an area that is 20 feet wide, simply plug 20 in to the function. $C(20) = 26(20) = 520$ dollars.

If the farmer wants to spend \$1300, we just set the cost function equal to 1300 and solve for w. Then, we must find the area. So, $26w = 1300$ and then $w = 50$. So the area he is able to enclose is $2w(w) = 2(50)(50) = 5000 sq.ft.$

Exercise 4:

A box is to be constructed that has a square bottom. Its height is twice the length of a side. Material for the sides costs

$\$0.10/cm^2$ *(square centimeter). Material for the top and bottom costs* $\$.15/cm^2$. *Write an equation for the cost of the box as a function of the length of a side. How much will it cost to build a box that is 5 cm on a side?*

Solution to #4:

Let a be a length of a side of the square bottom (or top). Then the (area of the top) = (area of the bottom) = $a^2 cm^2$. *The height of the box is twice the length of a side, so the height of the box is* $2a$. *So, the area of the other 4 sides are each* $(2a)(a) = 2a^2 cm^2$. *Taking into account the cost of the matieral, our function is now* $C(a) = (0.10)(4)(2a^2) + (0.15)(2)(a^2)$.

To build a box with $5cm$ side length, plug 5 into your function and you will get the cost. $C(5) = (0.10)(4)(2(5)^2) + (0.15)(2)(5^2) = 20 + 7.5 = 27.50$ *dollars.*

Cost/Revenue/Profit Functions

Exercise 5:

You are manufacturing a product that costs $\$5.00$ *per item to build. You also have fixed costs of* $\$1500$. *You sell the item for* $\$8.00$ *each for purchases up to 50 items. If the customer buys at least 50 items at a single sale you sell them for* $\$7.50$ *each. Write functions for cost, revenue and profit for a single sale.*

Solution to #5:

Below you'll see how to visualize the revenue function $R = R(q)$ (R means revenue, q means quantity) by combining the graphs of the following two revenue functions:

$R_1(q) = 8q$ *(you sell at* $\$8.00$ *per piece)*

$R_2(q) = 7.5q$ *(you sell at only* $\$7.50$ *per piece) Please note that I wrote $R1(q)$ and $R2(q)$ instead of $R_1(q)$ and $R_2(q)$ because I could not find an easy way to write subscripts in the figures below.*

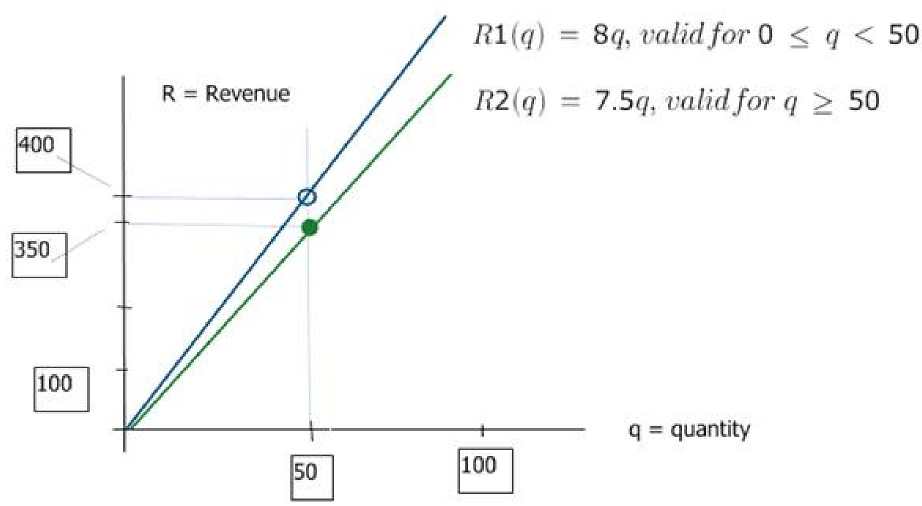
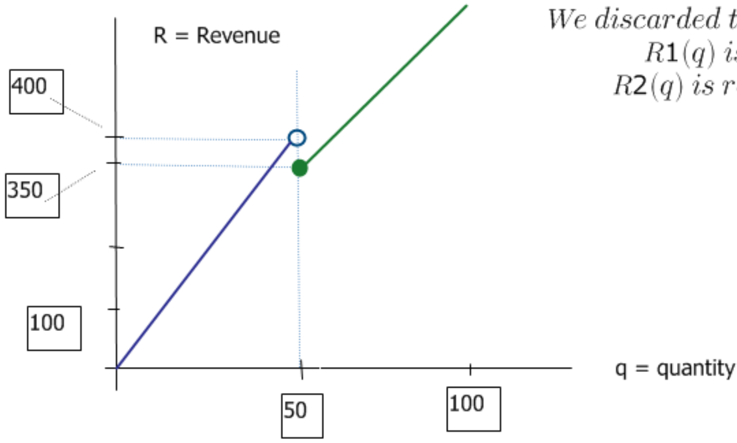


Figure 1: Both revenue functions



$We\ discarded\ the\ parts\ that\ are\ not\ allowed:$
$R1(q)\ is\ removed\ for\ q\ \geq\ 50,$
$R2(q)\ is\ removed\ for\ 0\ \leq\ q\ <\ 50.$

Figure 2: Combined revenue functions

$$\textit{cost is } 4\pi m^2 \times \$20/m^2 \ = \ \$80.00\pi \ \approx \ \$80 \times 3.14 \ = \ \$251.20$$

Exercise 6:

You are going to find 3 different functions for the revenue from the sale of 2 products.

a. *The revenue from the sale of product A is $\$6.00$ each. The revenue from product B is $\$9.00$ each. Write a function for the total revenue in terms of the quantities of the two products.*

b. *Suppose you only have a demand for a total of 108 products. Write a function for the revenue in terms of the number of product A that is sold (Hint: $B = 108 - A$).*

c. *Again, the demand is for a total of 108 products. Write a function for the revenue in terms of the number of product B that is sold.*

d. *Find the revenue from selling 48 of product A and 60 of product B using all three functions to demonstrate that they all give the same answer.*

Solution to #6a:

Let a be the number sold for Product A. Let b be the number sold for Product B. Then the function for revenue is $R = 6a + 9b$.

Solution to #6b:

Since your demand is 108, you know $a + b = 108$. This means that $b = 108 - a$. You can plug this into your original function; $R = 6a + 9(108 - a)$. This function is indeed in terms of the number of Product A sold, since a is the only variable.

Solution to #6c:

Same thing as Part (b), only now we solve for a, like this; $a = 108 - b$. Plug this in to get $R = 6(108 - b) + 9b$.

Solution to #6d:

Plug it in three different ways. First, from Part (a), $6(48) + 9(60) = 828$*. From Part (b),* $6(48) + 9(108 - 48) = 828$*. From Part (c),* $6(108 - 60) + 9(60) = 828$*. Check.*