Math 220 - Calculus f. Business and Management - Worksheet 3

Solutions for Worksheet 3 - Powers and Roots

Numerical Problems

Exercise 1:

Simplify each expression to a single number.

1a: $2^2 2^5$, *1b:* $(2^2)^3$, *1c:* $49^{1/2}$, *1d:* $27^{2/3}$, *1e:* 4^{-2},
1f: $36^{-1/2}$, *1g:* $(2^2+3^2)^2$, *1h:* $(-5)^2$, *1i:* -5^2, *1j:* $\sqrt{5^2+12^2}$

Solution to #1:

1a: $2^2 2^5 \ = \ 4 \times 32 \ = \ 128$
1b: $(2^2)^3 \ = \ 2^{2\times 3} \ = \ 2^6 \ = \ 64$
1c: $49^{1/2} \ = \ \sqrt{49} \ = \ 7$
1d: $27^{2/3} \ = \ (\sqrt[3]{27})^2 \ = \ 3^2 \ = \ 9$
1e: $4^{-2} \ = \ 1/4^2 \ = \ 1/16$
1f: $36^{-1/2} \ = \ 1/36^{1/2} \ = \ 1/\sqrt{36} \ = \ 1/6$
1g: $(2^2+3^2)^2 \ = \ (4+9)^2 \ = \ 13^2 \ = \ 169$
1h: $(-5)^2 \ = \ (-5)(-5) \ = \ 5^2 \ = \ 25$
1i: $-5^2 \ = \ -(5)^2 \ = \ -25$
1j: $\sqrt{5^2+12^2} \ = \ \sqrt{25+144} \ = \ \sqrt{169} \ = \ 13$

Algebra Problems

Exercise 2:

Simplify to x^r where r is a real number.

2a: $x^6\sqrt[3]{x}$, *2b:* $\sqrt{x}/x^4$, *2c:* $x^3\sqrt{x}/\sqrt[4]{x}$, *2d:* $(\sqrt{x})^3$, *2e:* $\sqrt[3]{x}^5$,
2f: $(1/x)^{2/3}$, *2g:* $(x^5)^3$, *2h:* x^{5^3}, *2i:* $\dfrac{x^{-1/2}}{x^3}$, *2j:* $(-x)^4$

Solution to #2:

2a: $x^6\sqrt[3]{x} \ = \ x^6x^{1/3} \ = \ x^{18/3+1/3} \ = \ x^{19/3}$
2b: $\sqrt{x}/x^4 \ = \ x^{1/2}x^{-4} \ = \ x^{1/2-4} \ = \ x^{-7/2}$
2c: $x^3\sqrt{x}/\sqrt[4]{x} \ = \ x^3x^{1/2}/x^{1/4} \ = \ x^{3+1/2-1/4} \ = \ x^{13/4}$
2d: $(\sqrt{x})^3 \ = \ (x^{1/2})^3 \ = \ x^{3/2}$
2e: $\sqrt[3]{x}^5 \ = \ x^{5\cdot 1/3} \ = \ x^{5/3}$
2f: $(1/x)^{2/3} \ = \ x^{-2/3}$
2g: $(x^5)^3 \ = \ x^{5\cdot 3} \ = \ x^{15}$
2h: $x^{5^3} \ = \ x^{(5^3)} \ = \ x^{125}$
2i: $\dfrac{x^{-1/2}}{x^3} \ = \ x^{-1/2}x^{-3} \ = \ x^{-1/2-3} \ = \ x^{-7/2}$
2j: $(-x)^4 \ = \ (-x)\cdot(-x)\cdot(-x)\cdot(-x) \ = \ x^4$

Domains

Exercise 3:

Find the domain of each function.

3a: $f(x) = \sqrt{3x+2}$, *3b:* $f(x) = \sqrt[3]{5x-6}$, *3c:* $f(x) = \sqrt{4-x}$,
3d: $f(x) = \sqrt{-7x}$, *3e:* $f(x) = \dfrac{2}{5x+4}$, *3f:* $f(x) = \dfrac{6}{3-7x}$,
3g: $f(x) = 5/(-6x)$, *3h:* $f(x) = \sqrt{x^2-x-6}$, *3i:* $f(x) = \dfrac{1}{\sqrt{2x-8}}$

Solution to #3:

3a: $f(x) = \sqrt{3x+2}$*: The radicand* $3x+2$ *cannot be negative. We require* $3x+2 \geq 0$*, i.e.,* $3x \geq -2$*, i.e.,* $x \geq -2/3$*, i.e.,* $D_f = [-2/3, \infty)$.

3b: $f(x) = \sqrt[3]{5x-6}$*:* $\sqrt[3]{A}$ *is defined for any number* A*, i.e.,* $D_f = \mathbb{R} = (-\infty, \infty)$.

3c: $f(x) = \sqrt{4-x}$*: same as 3A: We require* $4-x \geq 0$*, i.e.,* $4 \geq x$*, i.e.,* $D_f = (-\infty, 4]$.

3d: $f(x) = \sqrt{-7x}$*: same as 3A: We require* $-7x \geq 0$*, i.e.,* $7x \leq 0$*(!!), i.e.,* $x \leq 0$*, i.e.,* $D_f = (-\infty, 0]$.

3e: $f(x) = \dfrac{2}{5x+4}$*: Not allowed to divide by zero. We require* $5x+4 \neq 0$*, i.e.,* $x \neq -4/5$*, i.e.,* $D_f = \{x | x \neq -4/5\} = (-\infty, -4/5) \cup (-4/5, \infty)$.

3f: $f(x) = \dfrac{6}{3-7x}$*: Not allowed to divide by zero. We require* $3-7x \neq 0$*, i.e.,* $x \neq 3/7$*, i.e.,* $D_f = \{x | x \neq 3/7\} = (-\infty, 3/7) \cup (3/7, \infty)$.

3g: $f(x) = 5/(-6x)$*: Not allowed to divide by zero. We require* $5/(-6x) \neq 0$*, i.e.,* $x \neq 0$*, i.e.,* $D_f = \{x | x \neq 0\} = (-\infty, 0) \cup (0, \infty)$.

3h: $f(x) = \sqrt{x^2-x-6}$*: same as 3A: We require* $x^2-x-6 \geq 0$*. It' s easiest to factor the quadratic:* $x^2-x-6 = (x+2)(x-3)$*.* $(x+2)(x-3) \geq 0$ *means*
either a) both $(x+2) \geq 0$ *and* $(x-3) \geq 0$*, i.e., both* $x \geq -2$ *and* $x \geq 3$*, i.e.,* $x \geq 3$
or b) both $(x+2) \leq 0$ *and* $(x-3) \leq 0$*, i.e., both* $x \leq -2$ *and* $x \leq 3$*, i.e.,* $x \leq -2$.
a), b) together: $x \leq -2$ *or* $x \geq 3 \Rightarrow$ $D_f = (-\infty, -2) \cup (3, \infty)$.

3i: $f(x) = \dfrac{1}{\sqrt{2x-8}}$*: Not allowed to divide by zero. We require both*
a: $\sqrt{2x-8} \neq 0$*, i.e.,* $x \neq 4$.
b: Radicand $2x-8 \geq 0$*, i.e.,* $x \geq 4$.
a, b together: $x > 4$ *is required,i.e.,* $D_f = (4, \infty)$.