Math 220 - Calculus f. Business and Management - Worksheet 4

Solutions for Worksheet 4 - Polynomials and Fractions

Roots of Polynomials

Exercise 1: *Find the roots (zeroes) of each function. Note: sometimes the factors will be in radical form.*

$$\text{1a: } f(x) = x+7, \quad \text{1b: } f(x) = 3-2x, \quad \text{1c: } f(x) = x^2-x-20,$$
$$\text{1d: } f(x) = 6x^2+x-1, \quad \text{1e: } f(x) = x^2-4x+2, \quad \text{1f: } f(x) = x^3-5x^2+14x$$

Solution to #1:

1a: Simply set $x+7=0$ *and solve to get* $x=-7$.

1b: Set $3-2x=0$ *so that* $-2x=-3$ *and then* $x=\dfrac{3}{2}$.

1c: We factor $f(x) = x^2-x-20 = (x+4)(x-5)=0$, *i.e.,* $x=-4$ *or* $x=5$.

1d: We factor $f(x) = 6x^2+x-1 = (3x-1)(2x+1)=0$, *i.e.,* $x=4/12=1/3$ *or* $x=-6/12=-1/2$.

1d - Alternate solution: If you do not see quickly how to factor f(x), use the quadratic formula instead: $x = \dfrac{1}{2\cdot 6}\Big(-1 \pm \sqrt{1^2-4(6)(-1)}\Big) = 1/12(-1\pm\sqrt{25}) = 1/12(-1\pm 5) = \dfrac{-1\pm 5}{12}$, *i.e.,* $x=4/12=1/3$ *or* $x=-6/12=-1/2$.

1e: $f(x) = x^2-4x+2=0$ *means* $x=\dfrac{1}{2}\Big(-(-4)\pm\sqrt{(-4)^2-4(1)(2)}\Big) = 1/2(4\pm\sqrt{8}) = 1/2(4\pm 2\sqrt{2}) = 2\pm\sqrt{2}$, *i.e.,* $x=2+\sqrt{2}$ *or* $x=2-\sqrt{2}$.

1f: $f(x) = x^3-5x^2+14x = x(x^2-5x+14)=0$ *means* $x=0$ *or* $x^2-5x+14=0$. *It's fastest to use the Quadratic Formula:* $x=\dfrac{1}{2}\Big(-(-5)\pm\sqrt{(-5)^2-4(1)(14)}\Big) = 1/2(5\pm\sqrt{25-56}) = 1/2(5\pm\sqrt{-31})$. *There are no further solutions because the radicand is negative, but remember that you got the solution* $x=0$ *independently, so do not throw that one away:* $f(x)$ *is zero only if* $x=0$.

1f - Alternate solution: If you cannot remember the quadratic formula, you can still use completion of the square instead: $f(x) = x^3-5x^2+14x = x(x^2-5x+14)=0$ *means* $x=0$ *or* $x^2-5x+14=0$. *You can use the Quadratic Formula but I shall illustrate here the "completion of the square" technique:* $x^2-5x+14 = x^2-2(5/2)x+(5/2)^2+14-(5/2)^2$ $= (x-5/2)^2+14-25/4 = (x-5/2)^2+\dfrac{56-25}{4}=0$ *means the same as* $(x-5/2)^2 = -\dfrac{56-25}{4} = -31/4$. *But squares are never negative, so* $x^2-5x+14$ *is never zero:* $f(x)$ *is zero only if* $x=0$.

Domains of Functions

Exercise 2: *Find the domain of each function.*

$$a\text{: } f(x) = \frac{-6}{x^2+2x-24},\quad b\text{: } f(x) = \frac{5x}{2x^2+3x-7},\quad c\text{: } f(x) = \frac{x+2}{x^2+6x+8},$$
$$d\text{: } f(x) = \frac{8}{x^2+2x+5},\quad e\text{: } f(x) = \frac{1}{\sqrt{2x-8}},\quad f\text{: } f(x) = \frac{2x}{\sqrt{5-3x}},$$
$$g\text{: } f(x) = \frac{x^2}{\sqrt{-4x}},\quad h\text{: } f(x) = \frac{1}{\sqrt{x^2-3x-18}},\quad i\text{: } f(x) = \frac{\sqrt{x}}{x^2+x-6}$$

Solution to #2:

a: $f(x) = \dfrac{-6}{x^2+2x-24}$*: Not allowed to divide by zero. We factor the denominator* $x^2+2x-24 = (x+6)(x-4)$. *This is zero for* $x=-6, x=4$ *and we must disallow these values:*
$D_f = \{x|x \neq -6, x \neq 4\}$*; same as* $D_f = (-\infty,-6)\cup(-6,4)\cup(4,\infty)$.

b: $f(x) = \dfrac{5x}{2x^2+3x-7}$*: Not allowed to divide by zero. We use the quadratic formula:* $2x^2+3x-7$ *means* $x = \dfrac{1}{2(2)}\Big(-3 \pm \sqrt{3^2-4(2)(-7)}\Big) = 1/4(-3 \pm \sqrt{9+56} = 1/4(-3\pm\sqrt{65})$*, i.e.,* $x = (-3\pm\sqrt{65})/4$ *must be excluded from* D_f*:* $D_f = \{x|x \neq (-3-\sqrt{65})/4, x \neq (-3+\sqrt{65})/4\}$*; same as*
$D_f = (-\infty, \dfrac{-3-\sqrt{65}}{4}) \cup (\dfrac{-3-\sqrt{65}}{4}, \dfrac{-3+\sqrt{65}}{4}) \cup (\dfrac{-3+\sqrt{65}}{4}, \infty)$.

c: $f(x) = \dfrac{x+2}{x^2+6x+8}$*: Not allowed to divide by zero. We factor the denominator* $x^2+6x+8 = (x+2)(x+4)$. *This is zero for* $x=-2, x=-4$ *and we must disallow these values:*
$D_f = \{x|x \neq -4, x \neq -2\}$*; same as* $D_f = (-\infty,-4)\cup(-4,-2)\cup(-2,\infty)$.

Note that even though $f(x) = \dfrac{x+2}{(x+2)(x+4)}$ *simplifies to* $f(x) = \dfrac{1}{(x+4)}$ *and the "*$x=-2$ *division by zero problem" has magically disappeared, this does not change the fact that* -2 *does not belong to* D_f *and that* $f(-2)$ *is nonsensical!*

d: $f(x) = \dfrac{8}{x^2+2x+5}$*: It's fastest to use the Quadratic Formula:* $x = \dfrac{1}{2}\Big(-2\pm\sqrt{2^2-4(1)(5)}\Big) = 1/2(-2\pm\sqrt{-16})$

This can be solved by use of the Quadratic Formula. I shall use completion of the square instead. The denominator is zero if $x^2+2x+5 = x^2+2x+4+1 = 0$*, i.e.,* $(x+2)^2+1=0$*, i.e.,* $(x+2)^2 = -1$*. Squares will never be negative and we need not worry about dividing by zero. Nothing needs to be excluded from* D_f *and we get* $D_f = \mathbb{R} = (-\infty,\infty)$. *There are no further solutions because the radicand is negative and we need not worry about dividing by zero. Nothing needs to be excluded from* D_f *and we get* $D_f = \mathbb{R} = (-\infty,\infty)$.

d - Alternate solution: If you cannot remember the quadratic formula, you can still use completion of the square instead: The denominator is zero if $x^2+2x+5 = x^2+2x+4+1 = 0$*, i.e.,* $(x+2)^2+1=0$*, i.e.,* $(x+2)^2=-1$*. Squares will never be negative and we need not worry about dividing by zero, i.e.,* $D_f = \mathbb{R} = (-\infty,\infty)$.

e: We are not allowed to divide by zero or take the square root of a negative number. So, the inside of the square root can't be zero, and it cant be negative. In other words, $2x-8>0$*. So,* $2x>8$ *which means that* $x>4$*. Therefore,* $D_f = (4,\infty)$.

f: As in B5, we are not allowed to divide by zero or take the square root of a negative number. So, we need that $5-3x>0$*, which means* $-3x>-5$*, or* $x>\dfrac{5}{3}$*. Therefore,* $D_f = (\dfrac{5}{3},\infty)$.

g: $f(x) = \dfrac{x^2}{\sqrt{-4x}}$*: Two things to worry about: 1) we cannot divide by zero and 2) square roots cannot be negative.*
1) means we need $\sqrt{-4x} \neq 0$*; same as* $x \neq 0$*.*
2) means we need $-4x \geq 0$*, i.e.,* $4x \leq 0$ *(!), i.e.,* $x \leq 0$*.*
1) and 2) together means we must have $x < 0$*, i.e.,* $D_f = (-\infty, 0)$*.*

h: $f(x) = \dfrac{1}{\sqrt{x^2 - 3x - 18}}$*: Not allowed to divide by zero and radicand must be* ≥ 0*, i.e., need* $x^2 - 3x - 18 > 0$ *(STRICLY greater!). We factor* $x^2 - 3x - 18 = (x+3)(x-6)$ *and observe separately what happens for* $-\infty < x < -3$*,* $x = -3$*,* $-3 < x < 6$*,* $x = 6$*,* $6 < x < \infty$*:*

If x does the following:	*Then x+3 and x-6 do this:*	*Hence (x+3)(x-6) is:*
$-\infty < x < -3$	$x + 3 < 0$ *and* $x - 6 < 0$	> 0*: belongs to* D_f
$x = -3$	$x + 3 = 0$ *and* $x - 6 < 0$	$= 0$*: **NOT ALLOWED***
$-3 < x < 6$	$x + 3 > 0$ *and* $x - 6 < 0$	< 0*: **NOT ALLOWED!***
$x = 6$	$x + 3 > 0$ *and* $x - 6 = 0$	$= 0$*: **NOT ALLOWED***
$6 < x < \infty$	$x + 3 > 0$ *and* $x - 6 > 0$	> 0*: belongs to* D_f

In case you find it it easier, it's perfecty acceptable for your solutions if you draw a picture and pick some specific number in each one of the three intervals to illustrate what sign the product $(x+3)(x-6)$ *will have in each one of those segments:*

$x = -5$	$(-5+3)(-5-6) = (-2)(-11)$	> 0*: belongs to* D_f
$x = -3$	$(-3+3)(-3-6) = 0 \cdot (-9)$	$= 0$*: **NOT ALLOWED***
$x = 0$	$(0+3)(0-6) = 3(-9)$	< 0*: **NOT ALLOWED!***
$x = 6$	$(6+3)(6-6) = 9 \cdot 0$	$= 0$*: **NOT ALLOWED***
$x = 10$	$(10+3)(10-6) = 13 \cdot 4$	> 0*: belongs to* D_f

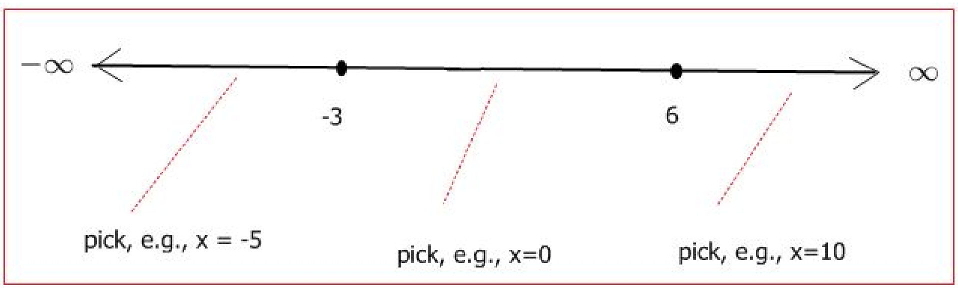

Figure 1: Five choices for the sign of $(x+3)(x-6)$

Either way it follows that the domain consists of the left and right end pieces: $D_f = (-\infty, -3) \cup (6, \infty)$*.*

i: $f(x) = \dfrac{\sqrt{x}}{x^2 + x - 6}$*: Not allowed to divide by zero and the radicand* x *must be* ≥ 0*.* $x^2 + x - 6 = (x+3)(x-2)$ *is zero for* $x = -3$ *or* $x = 2$*. All together: we must exclude* $x < 0, x = -3, x = 2$*, i.e.,* $D_f = [0, 2) \cup (2, \infty)$*.*