Math 220 - Calculus f. Business and Management - Worksheet 14

Solutions for Worksheet 14 - Find the derivative of each function

Hint: Before you use the product rule for a product and the quotient rule for a quotient, think for a moment whether a simple manipulation of the function allows you to use the power rule instead and save some work.

Exercise 1: *Find the derivative of each function*

$$\textbf{1a}: f(x) = x^3 + 2x - 5, \qquad \textbf{1b}: f(x) = 6x^4 - 3x^2 + 2x - 7, \qquad \textbf{1c}: f(x) = \sqrt[4]{x}.$$

Solution to #1:

For 1a: $\frac{d}{dx}(x^3 + 2x - 5) = 3x^2 + 2$. *We used the power rule.*

For 1b: $\frac{d}{dx}(6x^4 - 3x^2 + 2x - 7) = 24x^3 - 6x + 2$ *again using the power rule.*

For 1c: $\frac{d}{dx}(\sqrt[4]{x}) = \frac{d}{dx}(x^{1/4}) = \frac{1}{4}x^{-3/4} = \frac{1}{4\sqrt[4]{x^3}}$. *Here we change the radical into a more useful form, and then use the power rule.*

Exercise 2*:*

$$\textbf{2a}: f(x) = \frac{1}{x^3}, \qquad \textbf{2b}: f(x) = \sqrt{x^5}, \qquad \textbf{2c}: f(x) = \frac{7}{\sqrt{x}}, \qquad \textbf{2d}: f(x) = \sqrt[3]{\frac{5}{x^2}}.$$

Solution to #2:

For 2a: $\frac{d}{dx}\frac{1}{x^3} = \frac{d}{dx}x^{-3} = -3x^{-4} = \frac{-3}{x^4}$.

For 2b: $\frac{d}{dx}\sqrt{x^5} = \frac{d}{dx}x^{5/2} = \frac{5}{2}x^{3/2}$.

For 2c: $\frac{d}{dx}\frac{7}{\sqrt{x}} = \frac{d}{dx}7x^{-1/2} = 7(\frac{-1}{2})x^{-3/2} = \frac{-7}{2\sqrt{x^3}}$.

For 2d: $\frac{d}{dx}\sqrt[3]{\frac{5}{x^2}} = \frac{d}{dx}\frac{\sqrt[3]{5}}{\sqrt[3]{x^2}} = \frac{d}{dx}\frac{\sqrt[3]{5}}{x^{2/3}} = \frac{d}{dx}\sqrt[3]{5}x^{-2/3} = \sqrt[3]{5}(-\frac{2}{3})x^{-5/3} = -\frac{2}{3}\sqrt[3]{\frac{5}{x^5}}$

Exercise 3*:*

$$\textbf{3a}: f(x) = (5x+4)(9x+2), \qquad \textbf{3b}: f(x) = (3x^2 - 7x + 4)\cdot\frac{1}{x}, \qquad \textbf{3c}: f(x) = (8x^3 + 2)\sqrt{x}.$$

Solution to #3:

For 3a: We can use the product rule. So, $\frac{d}{dx}(5x+4)(9x+2) = 5(9x+2)+(5x+4)(9) = 45x+10+45x+36 = 90x+46$.

For 3b: It's simpler to multiply this out rather than using the product rule. So,

$$\frac{d}{dx}(3x^2 - 7x + 4)\cdot\frac{1}{x} = \frac{d}{dx}(3x - 7 + 4x^{-1}) = 3 + 4(-1)x^{-2} = 3 - \frac{4}{x^2}$$

For 3c: It's simpler to multiply this out rather than using the product rule. So,

$$\frac{d}{dx}(8x^3+2)(\sqrt{x}) \;=\frac{d}{dx}(8x^{7/2}+2x^{1/2}) \;=\; 8(7/2)x^{5/2}+x^{-1/2} \;=\; 28x^{5/2} \;+\; \frac{1}{\sqrt{x}}$$

Exercise 4:

$$\textbf{\textit{4a}}: f(x) \;=\; \big(\frac{1}{3x^4}+x\big)\,(2-\sqrt[4]{5x}+x^2), \qquad \textbf{\textit{4b}}:\; f(x) \;=\; x(4x^5+7).$$

Solution to #4:

For 4a:

$$\begin{aligned}\frac{d}{dx}\big(\frac{1}{3x^4}+x\big)\,(2-\sqrt[4]{5x}+x^2) &= \frac{d}{dx}\big(\frac{1}{3}x^{-4}\;+x\big)\,(2-(5x)^{1/4}+x^2)\\ &= (-\frac{4}{3}x^{-5}+1)(2-(5x)^{1/4}+x^2)+\big(\frac{1}{3}x^{-4}\;+x\big)(\frac{1}{4}(5x)^{-3/4}+2x)\end{aligned}$$

For 4b: Here we could use the product rule, but it is easier to distribute x through and go from there.

$$\frac{d}{dx}x(4x^5+7) \;=\; \frac{d}{dx}4x^6+7x \;=\; 24x^5+7$$

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Exercise 5:

$$\textbf{\textit{5a}}: f(x) \;=\; \frac{2x+5}{3x-7}, \qquad \textbf{\textit{5b}}:\; f(x) \;=\; \frac{5x^2-7x+2}{x^3-9}, \qquad \textbf{\textit{5c}}:\; f(x) \;=\; \frac{4x^4-7x^2}{x}.$$

Solution to #5*:*

For 5a: Quotient rule.

$$\frac{d}{dx}\frac{2x+5}{3x-7} \;=\; \frac{2(3x-7)-(2x+5)(3)}{(3x-7)^2}$$

For 5b: Quotient rule again.

$$\frac{d}{dx}\frac{5x^2-7x+2}{x^3-9} \;=\; \frac{(x^3-9)(10x-7)-(5x^2-7x+2)(3x^2)}{(x^3-9)^2}$$

For 5c: You may use the quotient rule, however it is easier to just divide that x out and go from there. So,

$$\frac{d}{dx}\frac{(4x^4-7x^2)}{x} \;=\; \frac{d}{dx}(4x^3-7x) \;=\; 12x^2-7$$

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Exercise 6:

$$\textbf{\textit{6a}}: f(x) \;=\; \frac{6}{-6x^2+8x+12}, \qquad \textbf{\textit{6b}}:\; f(x) \;=\; \frac{6x-7}{9+\sqrt{3x}}, \qquad \textbf{\textit{6c}}:\; f(x) \;=\; \frac{\sqrt{x}}{\sqrt[3]{x}}.$$

Solutions to #6

For 6a: Quotient Rule. So,

$$\frac{d}{dx}\frac{6}{-6x^2+8x+12} = \frac{(-6x^2+8x+12)(0)-6(-12x+8)}{(-6x^2+8x+12)^2} = \frac{72x-48}{(-6x^2+8x+12)^2}$$

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For 6b: Quotient Rule. So,

$$\begin{aligned}\frac{d}{dx}\Big(\frac{6x-7}{9+\sqrt{3x}}\Big) = \frac{d}{dx}\Big(\frac{6x-7}{9+\sqrt{3}x^{1/2}}\Big) &= \frac{(9+\sqrt{3}x^{1/2})(6) \;-\; (6x-7)(1/2)\sqrt{3}x^{-1/2}}{(9+(3x)^{1/2})^2}\\ &= \frac{(9+\sqrt{3x})(6) \;-\; (6x-7)(\sqrt{3}/(2\sqrt{x})}{(9+\sqrt{3x})^2}\end{aligned}$$

For 6c: it is easier to simplify. So,

$$\frac{d}{dx}\frac{\sqrt{x}}{\sqrt[3]{x}} = \frac{d}{dx}\frac{x^{1/2}}{x^{1/3}} = \frac{d}{dx}x^{1/6} = \frac{1}{6}x^{-5/6}$$

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