# Math 220 - Calculus f. Business and Management - Worksheet 16

## *Solutions for Worksheet 16 - Exponents, Logs and the Chain Rule*

### *Find the derivative of each function*

*Hint: Some functions are much easier to differentiate if you use your knowledge of lessons 5 and 6 on exponents of logs to write them differently. Examples are A3 and D3.*

***Exercise 1:***

$$\textbf{\textit{1a}}: f(x) = 5^x, \qquad \textbf{\textit{1b}}: f(x) = \log_3 x, \qquad \textbf{\textit{1c}}: f(x) = \ln(x^2), \qquad \textbf{\textit{1d}}: f(x) = \log x.$$

***Solution to #1****:*

*For 1a:* $\dfrac{d}{dx}(5^x) = 5^x \ln(5)$.

*For 1b:* $\dfrac{d}{dx}\log_3 x = \dfrac{1}{x\ln 3}$

*For 1c:* $\dfrac{d}{dx}\ln(x^2) = \dfrac{d}{dx}(2\ln(x)) = 2\cdot\dfrac{1}{x} = \dfrac{2}{x}$*. Here we used a rule of logarithms in the first step.*

*For 1d:* $\dfrac{d}{dx}\log(x) = \dfrac{1}{x\ln 10}$.

***Exercise 2****:*

$$\textbf{\textit{2a}}: f(x) = 7^x\,(x^2+2x-5), \qquad \textbf{\textit{2b}}: f(x) = (4x+2)\,\log_3 x, \qquad \textbf{\textit{2c}}: f(x) = (1/x^4)\,\ln x.$$

***Solution to #2****: Each problem here uses the product rule.*

*For 2a:* $\dfrac{d}{dx}7^x(x^2+2x-5) = (7^x)(\ln(7))(x^2+2x-5) + (7^x)(2x+2)$.

*For 2b:* $\dfrac{d}{dx}(4x+2)\log_3 x = (4)(\log_3 x) + (4x+2)(\dfrac{1}{x\ln 3})$.

*For 2c:* $\dfrac{d}{dx}(1/x^4)\,\ln x = \dfrac{d}{dx}(x^{-4})\,\ln x = (-4x^{-5})(\ln x) + (x^{-4})\left(\dfrac{1}{x}\right) = \dfrac{-4\ln x + 1}{x^5}$.

***Exercise 3****:*

$$\textbf{\textit{3a}}: f(x) = \frac{\log_3 x}{5x^3+2x+10}, \qquad \textbf{\textit{3b}}: f(x) = \frac{x^3+4x-7}{6^x}, \qquad \textbf{\textit{3c}}: f(x) = \frac{x+\ln x}{2^x}.$$

***Solution to C****:*

*Each problem here uses the quotient rule.*

*For 3a:* $\dfrac{d}{dx}\dfrac{\log_3 x}{5x^3+2x+10} = \dfrac{(5x^3+2x+10)(\dfrac{1}{x\ln 3}) - (\log_3 x)(15x^2+2)}{(5x^3+2x+10)^2}$.

*For 3b:* $\dfrac{d}{dx}\dfrac{x^3+4x-7}{6^x} = \dfrac{(6^x)(3x^2+4) - (x^3+4x-7)(6^x)(\ln 6)}{6^{2x}}$.

*For 3c:* $\dfrac{d}{dx}\dfrac{x+\ln x}{2^x} = \dfrac{(2^x)(1+(1/x)) - (x+\ln x)(2^x)(\ln 2)}{x^{2x}}$.

***Exercise 4***:

$$\textbf{\textit{4a}} : f(x) = 6^{3x^5-5x^3+7}, \qquad \textbf{\textit{4b}} : f(x) = \ln(x^2),$$
$$\textbf{\textit{4c}} : f(x) = (\log_4 x)^3, \qquad \textbf{\textit{4d}} : f(x) = \ln(x^4 - 3x^2 + 2x).$$

*Note that 4b is the same as 1c above. This time use the chain rule.*

***Solution to #4***:

*We will use the chain rule in each problem. Notice that D2 is the same as A3, however the method we use here will be different.*

*For 4a: Let* $f(u) = 6^u$ *and let* $u(x) = 3x^5 - 5x^3 + 7$*. Then, using the chain rule,*

$$\frac{d}{dx}6^{3x^5-5x^3+7} = (6^{3x^5-5x^3+7})(\ln 6)(15x^4 - 15x^2)$$

*For 4b: Let* $f(u) = \ln(u)$ *and let* $u(x) = x^2$*. Then*

$$\frac{d}{dx}\ln(x^2) = (\frac{1}{x^2}\cdot 2x) = \frac{2}{x}$$

*Notice how this is the same answer as in A3.*

*For 4c: Let* $f(u) = u^3$ *and let* $u(x) = \log_4 x$*. Then,*

$$\frac{d}{dx}(\log_4 x)^3 = 3(\log_4(x))^2 \cdot \left(\frac{1}{x\ln 4}\right)$$

*For 4d: Let* $f(u) = \ln(u)$ *and let* $u(x) = x^4 - 3x^2 + 2x$*. Then,*

$$\frac{d}{dx}\ln(x^4 - 3x^2 + 2x) = \left(\frac{1}{x^4 - 3x^2 + 2x}\right)\cdot(4x^3 - 6x + 2)$$

***Exercise 5***:

$$\textbf{\textit{5a}} : f(x) = \log_3\sqrt{x}, \qquad \textbf{\textit{5b}} : f(x) = 4^{1/x}, \qquad \textbf{\textit{5c}} : f(x) = \log_5 e^{4x^2}.$$

***Solution to #5***: *Each problem uses the chain rule.*

*For 5a: Let* $f(u) = log_3 u$ *and let* $u(x) = \sqrt{x} = x^{1/2}$*. Then, using the chain rule,*

$$\frac{d}{dx}\log_3\sqrt{x} = \frac{1}{\sqrt{x}(\ln 3)}\cdot(\frac{1}{2}x^{-1/2})$$

*For 5b: Let* $f(u) = 4^u$ *and let* $u(x) = \dfrac{1}{x} = x^{-1}$*. Then,*

$$\frac{d}{dx}4^{1/x} = \frac{d}{dx}4^{x^{-1}} = 4^{x^{-1}}\cdot(\ln 4)(-x^{-2})$$

*For 5c: Let* $f(u) = \log_5 u$ *and let* $u(x) = e^{4x^2}$*. Then, apply the chain rule once more,*

$$\frac{d}{dx}\log_5 e^{4x^2} \;=\; (\frac{1}{e^{4x^2}(\ln 5)} \cdot (e^{4x^2}) \cdot (8x) \;=\; \frac{8x}{\ln 5}$$