Math 220 - Calculus f. Business and Management - Worksheet 19

Solutions for Worksheet 19 - Marginal Economic Values

Marginal cost, revenue and profit

Exercise 1: *The cost for rent, machinery etc is $\$50,000$ per year to make a product. Raw material and labor for the product costs $\$4.00$ per item. Write an equation for cost as a function of the number of items produced. Then write the marginal cost function. Estimate the cost increase by going from 100 units to 101 units. You should see that the estimate is equal to the actual cost increase. Can you explain why?*

Solution for #1:

The cost function is $C(x) = 50,000 + 4x$. So, the marginal cost is $C'(x) = 4$.

$$\begin{aligned} C(101) - C(100) &= \big[50,000 + 4(101)\big] - \big[50,000 + 4(100)\big] = 4(101 - 100) = 4, \textit{ i.e.,} \\ C(101) &= C(100) + C'(100) \cdot 1 \end{aligned}$$

and that last line tells us that the cost $C(101)$ for one more unit exactly amounts to the estimate which is $C(100)$ plust the marginal cost $C'(100)$ for 100 items.

Exercise 2: *The demand function for a product is $x = 1500 - 1.5p$ where x is the quantity produced and p is the price charged for the item. The cost function is $C(x) = 200x + 25000$.*

a) Write cost, revenue and profit as a function of price. Hints: Cost will be a composite function, revenue is quantity times price and profit is revenue minus cost.

Solution for #2a:

The cost function is

$$C(p) = 200(1,500 - 1.5p) + 25,000 = 325,000 - 300p,$$

the revenue function is

$$R(p) = p(1,500 - 1.5p) = 1,500p - 1.5p^2,$$

and the profit function is

$$P(p) = R(p) - C(p) = 325,000 - 1,800p + 1.5p^2.$$

b) Find the functions for marginal cost, revenue and profit.

Solution for #2b:

The marginals are the derivatives of C(p), R(p), P(p):

$$\begin{aligned} C'(p) &= \frac{d}{dp}(325,000 - 300p) = \boxed{-300} \\ R'(p) &= \frac{d}{dp}(1,500p - 1.5p^2) = \boxed{1,500 - 3p} \\ P'(p) &= R'(p) - C'(p) = 1,500 - 3p - (-300) = \boxed{1,800 - 3p} \end{aligned}$$

Exercise 3: *For this problem, the demand function is $X(p) = 6,000 - 10p$ and cost as a function of demand is $500 + 4x$.*

a) Write cost, revenue and profit as functions C(p), R(p), P(p) of price.

Solution for 3a:

We substitute $x = 6,000 - 10p$ *and obtain cost as a function of price:*

$$C(p) \;= 500 + 4(6,000 - 10p) \;=\; \boxed{24,500 - 40p}$$

Revenue is demand $\times$ *price:*

$$R(p) \;= (6,000 - 10p)p \;=\; \boxed{6,000p - 10p^2}$$

Profit is revenue minus cost:

$$P(p) \;= R(p) - C(p) \;=\; (6,000p - 10p^2) - (24,500 - 40p) \;=\; \boxed{-10p^2 + 6,040p - 24,500}$$

b) Find the functions for marginal cost, revenue and profit (depending on price).

Solution for 3b:

The marginals are the derivatives of C(p), R(p), P(p):

$$\begin{aligned} C'(p) \;&= \frac{d}{dp}(24,500 - 40p) \;=\; \boxed{-40} \\ R'(p) \;&= \frac{d}{dp}(6,000p - 10p^2) \;=\; \boxed{6,000 - 20p} \\ P'(p) \;&= R'(p) - C'(p) \;=\; \boxed{6,040 - 20p} \end{aligned}$$

c) Estimate how much the profit will change if the price is $200.00 *and is increased by* $1.00. *What if the price is* $400.00 *and the price is increased by* $1.00?

Solution for 3c:

We are asked to compute the marginal profit $P'(p)$ *at* 200 *and* 400:

$$\begin{aligned} P'(200) \;&= 6,040 - 20 \cdot 200 \;=\; 6,040 - 4,000 \;=\; \boxed{\$2,040.00} \\ P'(400) \;&= 6,040 - 20 \cdot 400 \;=\; 6,040 - 8,000 \;=\; \boxed{-\$1,960.00} \end{aligned}$$

We note that profit increases at a price of $200.00 *and decreases at a price of* $400.00.

d) At what prices is the profit increasing (marginal profit > 0*)? decreasing?*

Solution for 3d:

$$\begin{aligned} \textit{Profit increases at } p \textit{ means } \; P'(p) \;&>\; 0 \;\;\Rightarrow\;\; 6,040 - 20p \;>\; 0 \;\;\Rightarrow\;\; 6,040 \;>\; 20p \;\;\Rightarrow\;\; \boxed{p \;<\; 302} \\ \textit{Profit decreases at } p \textit{ means} P'(p) \;&<\; 0 \;\;\Rightarrow\;\; 6,040 - 20p \;<\; 0 \;\;\Rightarrow\;\; 6,040 \;<\; 20p \;\;\Rightarrow\;\; \boxed{p \;>\; 302} \end{aligned}$$

We note that profit increases as long as price is below the critical value of $302.00 *and increases if price exceeds that value. We also note that the results of part c) are consistent with our finding.*

Profit keeps increasing as the price goes up to $302.00 *and decreases thereafter. It is clear that* $p_0 = \$302.00$ *maximizes profit. You should see that the marginal profit is zero at that price point. This is an illustration of the general fact that* *if a function $f(x)$ achieves a maximun at a point x_0 then it's derivative $f'(x_0)$ will necessarily be zero. You will learn this and more about optimization in later lectures.*

e) Based on c and d, what do you think the price should be set at? How much profit will be made at this point (use your calculator)?

Solution for 3e:

Maximum profit is attained at a price of $\$302.00$. *It's actual value is*

$$P(302) \;=\; -\,10 \cdot 302^2 + 6,040 \cdot 302 - 24,500 \;=\; \boxed{\$887,540.00}$$