Math 220 - Calculus f. Business and Management - Worksheet 26

Solutions for Worksheet 26 - Interpreting Graphs

Given are the graphs of three functions 1) $y = f(x)$, 2) $y = g(x)$, 3) $y = h(x)$:

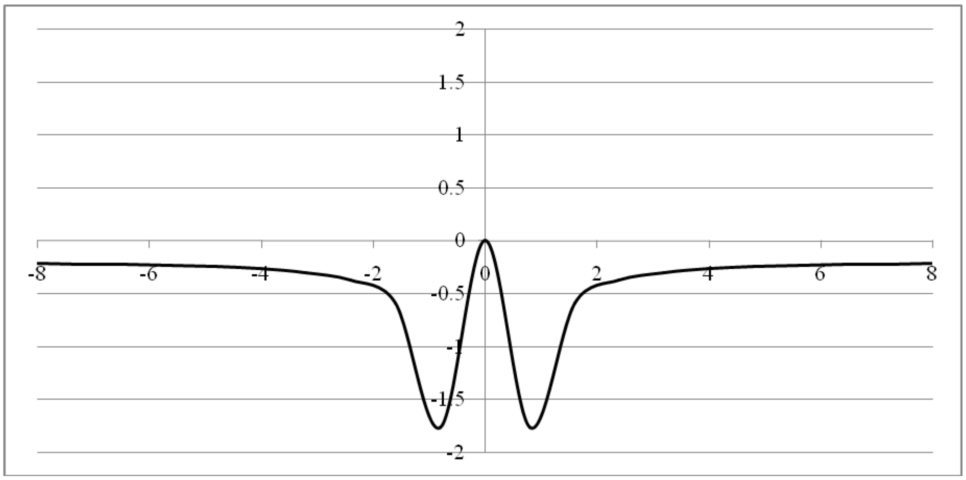

Figure 1: $y = f(x)$

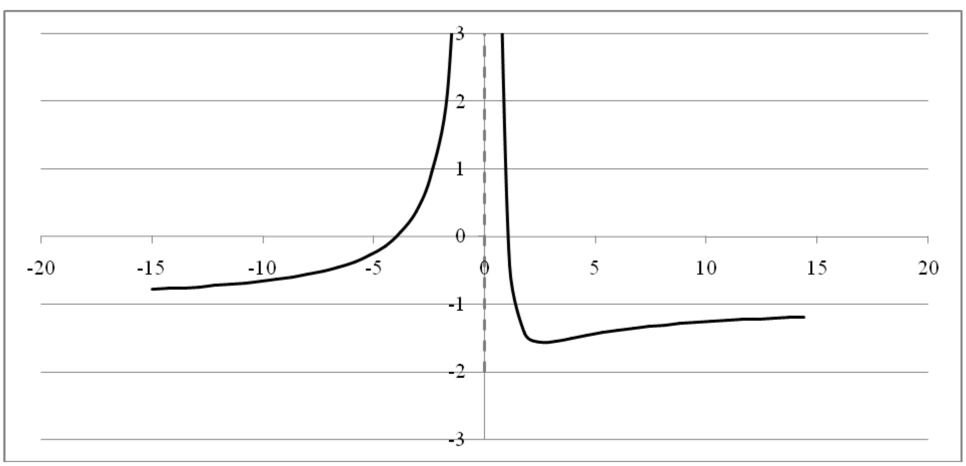

Figure 2: $y = g(x)$

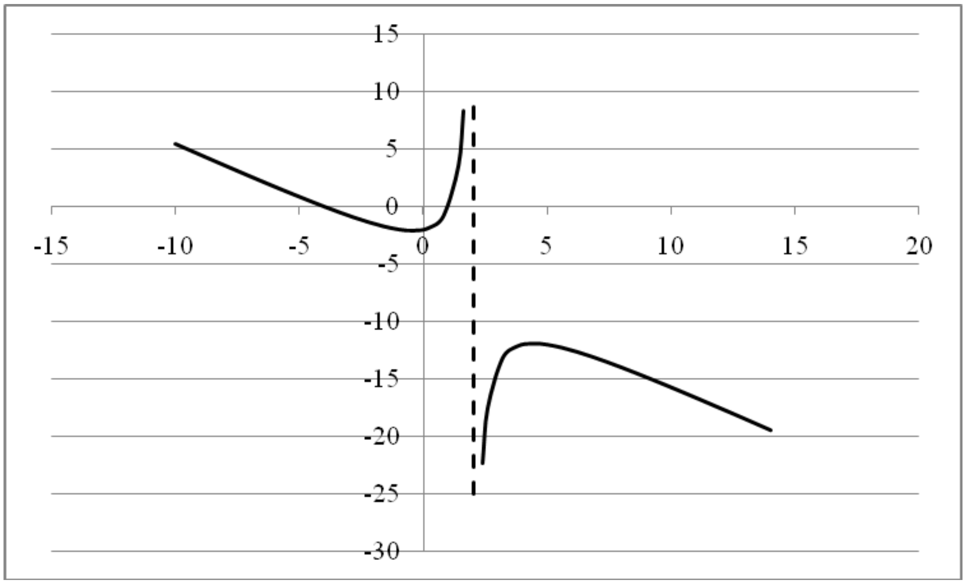

Figure 3: $y = h(x)$

Look at each graph and identify each of these items:

The domain of the function
Equations of all asymptotes to the graph of the function
The interval (or intervals) where the function is increasing and those where it is decreasing
The interval (or intervals) where the function is concave up and those where it is concave down
The critical points and critical values of the function that is graphed
The inflection points and their y-values on the graph of the function
The interval (or intervals) on which $f(x) > 0$
The interval (or intervals) on which $f'(x) > 0$
The interval (or intervals) on which $f''(x) > 0$

The solutions will give you specific values for x and y but you should think of them as approximate values, especially where change of concavity is concerned, as the drawings are imprecise. Hence, be not concerned if your answers differ slightly from those given in the solutions.

***Solution for graph 1**: $y = f(x)$*

a.) *The domain of f is $D_f = \mathbb{R}$.*
b.) *Horizontal asymptote: $y = 0$. Vertical asymptotes: NONE.*
c.) *$f(x)$ is increasing on $[-1,\ 0] \cup [1,\ \infty)$ and is decreasing on $(-\infty,\ -1] \cup [0, 1]$.*
d.) *$f(x)$ is concave up on $(-1.5,\ -0.5) \cup (0.5,\ 1.5)$ and is concave down on $(-\infty,\ -1.5) \cup (1.5,\ \infty)$.*
e.) *The critical points of f are -1, 0, and 1, and 1. The critical values of f are $f(-1) = -1.75$, $f(0) = 0$, and $f(1) = -1.75$.*
f.) *The inflection points and their function values are $(-1.5,\ -1.25),\ (1.5,\ -1.25),\ (-.5,\ -1.25)$ and $(.5,\ -1.25)$.*
g.) *$f(x) > 0$ NOWHERE.*
h.) *$f'(x) > 0$ on $(-1,\ 0) \cup (1,\ \infty)$.*
i.) *$f''(x) > 0$ on $(-1.5,\ -0.5) \cup (0.5,\ 1.5)$.*

Solution for graph 2: $y = g(x)$

a.) *The domain of* g *is* $D_g = \{ x : x \neq 0 \} = (-\infty, 0) \cup (0, \infty)$.
b.) *Horizontal asymptote:* $y = -1$. *Vertical asymptote:* $x = 0$.
c.) $g(x)$ *is increasing on* $(-\infty, 0]$ *and on* $[2.5, \infty)$. $g(x)$ *is decreasing on* $[0, 2.5)$.
d.) $g(x)$ *is concave up on* $(-\infty, 0) \cup (0, 5)$ *and is concave down on* $(5, \infty)$.
e.) *There is a critical point at* 2. *Its critical value is* $g(2) = -3$.
f.) *The function has an inflection point at* $x = 5$. *Its* y*-value is* -1.5.
g.) $g(x) > 0$ *on* $(-4, 0) \cup (0, 1)$.
h.) $g'(x) > 0$ *on* $(-\infty, 0)$.
i.) $g''(x) > 0$ *on* $(-\infty, 0) \cup (0, 5)$.

Solution for graph 3: $y = h(x)$

a.) *The domain of* h *is* $D_h = \{ x : x \neq 2 \} = (-\infty, 2) \cup (2, \infty)$.
b.) *Horizontal asymptotes: NONE. Vertical asymptote:* $x = 2$.
c.) $h(x)$ *is increasing on* $[0, 2) \cup (2, 4]$. $h(x)$ *is decreasing on* $(-\infty, 0] \cup [4, \infty)$.
d.) $h(x)$ *is concave up on* $(-\infty, 2)$ *and is concave down on* $(2, \infty)$.
e.) *The critical points of* h *are* 0 *and* 4 *with corresponding critical values* $h(0) = -2$ *and* $h(4) = -12$.
f.) *The graph of* h *has no inflection points as concavity only changes at the vertical asymptote.*
g.) $h(x) > 0$ *on* $(-\infty, -4) \cup (1, 2)$.
h.) $h'(x) > 0$ *on* $(0, 2) \cup (2, 4)$.
i.) $h''(x) > 0$ *on* $(-\infty, 2)$.