Math 220 - Calculus f. Business and Management - Worksheet 29

Solutions for Worksheet 29 - Elasticity

Exercise 1: *The demand function for a particular product is $\sqrt{50-p^2}$. Find out if the demand is elastic or inelastic at a price of \$3.00. Does this mean the revenue will go up or go down if the price is increased slightly?*

Solution to** 1**:

$$\begin{aligned}
\textit{Demand } q &= Q(p) \;=\; (50-p^2)^{1/2} \;\rightsquigarrow\; q' \;=\; \frac{dq}{dp} \;=\; (1/2)(-2)p(50-p^2)^{1/2} \;=\; \frac{-\,p}{\sqrt{50-p^2}}; \\
\textit{Elasticity } E(p) &= -\,\frac{dq}{dp}\cdot\frac{p}{q} \;=\; \frac{+\,p}{\sqrt{50-p^2}}\cdot\frac{+\,p}{\sqrt{50-p^2}} \;=\; \frac{p^2}{50-p^2} \quad (\textit{for } 0<p<\sqrt{50}) \\
\rightsquigarrow\;\; E(3) &= 9/41 \;<\; 1/4 \;<\; 1 \;\rightsquigarrow\; \textit{demand is very inelastic}\,.
\end{aligned}$$

We conclude that revenue goes up when the price is increased not only slightly: It will do so until elasticity has gone up to 1*.*

Exercise 2: *The demand function is $q(p) = 300e^{-.04p}$. Use the elasticity function to find the price for this product that generates the highest revenue.*

Solution to** 2**:

$$\begin{aligned}
\textit{Demand } q &= Q(p) \;=\; 300e^{-.04p} \;\rightsquigarrow\; q' \;=\; \frac{dq}{dp} \;=\; (-.4)(300)e^{-.04p} \;=\; -.4\cdot q \\
\rightsquigarrow\;\; \textit{Elasticity } E(p) &= -\,\frac{dq}{dp}\cdot\frac{p}{q} \;=\; \frac{+\,.4qp}{q} \;=\; .4p\,.
\end{aligned}$$

We find the highest revenue by solving $E(p)=1$ for p:

$$E(p) \;=1 \;\rightsquigarrow\; (4/10)p \;=\; 1 \;\rightsquigarrow\; p \;=\; 10/4 \;=\; \$2.50\,.$$

Exercise 3: *Using the situation in problem 2, if the price is currently \$20.00 will a slight increase in price result in higher or lower revenue?*

Solution to** 3**:

Because $E(p) = .4p$ (see problem 2), $E(20) \;=\; .4\cdot 20 \;=\; 8$. Demand is extremely elastic: Increasing the price slightly will lead to a loss of revenue. The price must be decreased to \$2.50 at which elasticity is 1 and we reach optimum revenue.

Exercise 4: *The demand for a product as a function of its price can be expressed as $D(p) = 675 - 0.25p^2$. Find the price which generates the highest revenue two ways. (Using the derivative of revenue and using the elasticity function). Note that if you get two different answers you have made a mistake somewhere.*

Solution 1 to** $\#4$**:

Find maximum revenue using elasticity:

$$\begin{aligned}
\textit{Demand } q &= D(p) \;=\; 675 - p^2/4 \;\rightsquigarrow\; q' \;=\; \frac{dq}{dp} \;=\; -p/2 \\
\rightsquigarrow\;\; \textit{Elasticity } E(p) &= -\,\frac{dq}{dp}\cdot\frac{p}{q} \;=\; \frac{+\,p}{2}\cdot\frac{p}{q} \;=\; \frac{p^2}{2q} \;=\; \frac{+\,p}{2}\cdot\frac{p}{q} \;=\; \frac{p^2}{2q}
\end{aligned}$$

We plug in $q = 675 - (1/4)p^2 = (2,700 - p^2)/4$:

$$E(p) = \frac{p^2}{2\Big((2,700 - p^2)/4\Big)} = \frac{p^2}{(2,700 - p^2)/2} = \boxed{\frac{2p^2}{2,700 - p^2}}$$

We find the highest revenue by solving $E(p) = 1$ *for* p:

$$E(p) = 1 \rightsquigarrow 2,700 - p^2 = 2p^2 \rightsquigarrow 3p^2 = 2,700 \rightsquigarrow p^2 = 900 \rightsquigarrow p = \pm 30\,.$$

No such thing as a negative price and we have maximum revenue at $\boxed{p = \$30.00}$.

Solution 2 to $\#4$***:***

Find maximum revenue by examining the critical points of the revenue function $r = R(p)$:

$$\text{Revenue } r = R(p) = pq = 675p - p^3/4 \rightsquigarrow r' = \frac{dr}{dp} = 675 - 3p^2/4$$

To find the critical points we solve $r' = 0$ *for* p:

$$r' = 0 \rightsquigarrow 675 - 3p^2/4 = 0 \rightsquigarrow 4 \cdot 675 = 3p^2 \rightsquigarrow 2,700 = 3p^2 \rightsquigarrow p^2 = 900 \rightsquigarrow p = \pm 30\,.$$

This last equation should look very familiar to you from solution 1. Again we obtain $\boxed{p = \$30.00}$ *as the price point for maximum revenue, once you know that there is indeed a max at* $p = \$30.00$. *For that compute the* 2nd *derivative:*

$$r" = \frac{dR'}{dp} = \frac{d}{dp}(675 - 3p^2/4) = (-3/2)p\,.$$

This is < 0 *for* $p = 30$ *and we have infact maximum revenue at that price point.*

Summary Both solution 1 and solution 2 are equal to \$30.00.