Math 220 - Calculus f. Business and Management - Worksheet 32

Solutions for Worksheet 32 - Integration by Substitution

Exercise 1: *Find the derivatives of each of these functions*

$$\textbf{1a)}: f(x) = e^{3x^2+2x} \qquad \textbf{1b)}: f(x) = (x^2+3x)^{27}$$

Solution to $1a$***:***

$$\textit{Chain rule:} \quad \frac{d}{dx}(e^{3x^2+2x}) = \boxed{(6x + 2) \cdot e^{3x^2+2x}}$$

Solution to $1b$***:***

$$\textit{Chain rule:} \quad \frac{d}{dx}(x^2+3x)^{27} = \boxed{(2x + 3) \cdot 27(x^2+3x)^{26}}$$

Exercise 2: *Use what you have seen in problem 1 to set up integration by substitution for the following:*

$$\textbf{2a)}: \int (6x+2)e^{3x^2+2x}\,dx \quad \textbf{2b)}: \int 5(2x+3)(x^2+3x)^{26}\,dx$$

Solution to $2a$***:***

$$u(x) = 3x^2 + 2x \Rightarrow \frac{du}{dx} = 6x + 2 \Rightarrow du = (6x + 2)dx$$
$$\Rightarrow \int (6x + 2)e^{3x^2+2x}\,dx = \int e^u\,du = e^u + C = \boxed{e^{3x^2+2x} + C}$$

Solution to $2b$***:***

$$u(x) = x^2 + 3x \Rightarrow \frac{du}{dx} = 2x + 3 \Rightarrow du = (2x + 3)dx$$
$$\Rightarrow \int 5(2x + 3)(x^2 + 3x)^{26}\,dx = \int u^{26}\,(5du) = 5\int u^{26}\,du = (5/27)u^{27} + C = \boxed{(5/27)(x^2 + 3x)^{27} + C}$$

Exercise 3: *Solve these integrals using substitution:*

$$\textbf{3a)}: \int (3x+2)^4\,dx \quad \textbf{3b)}: \int t\,e^{3t^2}\,dt \quad \textbf{3c)}: \int 2x\sqrt{5x^2-2}\,dx \quad \textbf{3d)}: \frac{4x^5}{x^6-8}dx$$
$$\textbf{3e)}: \int x(x-2)^5\,dx \quad \textbf{3f)}: \int e^{5t}\,dt$$

Solution to 3a:

$$u(x) = 3x + 2 \Rightarrow \frac{du}{dx} = 3 \Rightarrow du = 3dx \Rightarrow dx = (1/3)du$$
$$\Rightarrow \int (3x+2)^4\,dx = \int u^4\,\frac{du}{3} = \frac{1}{3}\int u^4\,du = \frac{1}{3}\cdot\frac{1}{5}u^5 + C = \boxed{\frac{(3x+2)^5}{15} + C}$$

Solution to 3b:

$$u(t) \;= 3t^2 \;\Rightarrow\; \frac{du}{dt} \;=\; 6t \;\;\Rightarrow\;\; du \;=\; 6t\,dt \;\;\Rightarrow\;\; dt \;=\; \frac{du}{6t}$$
$$\Rightarrow\; \int t\,e^{3t^2}\,dt \;=\; \int t\,e^u\,\frac{du}{6t} \;=\; \frac{1}{6}\int e^u\,du \;=\; \frac{1}{6}\cdot e^u \;+\; C \;=\; \boxed{\frac{1}{6}\cdot e^{3t^2} \;+\; C}$$

Solution to 3c:

$$u(x) \;= 5x^2 \;-\; 2 \;\;\Rightarrow\;\; \frac{du}{dx} \;=\; 10x \;\;\Rightarrow\;\; \frac{du}{5} \;=\; (2x)dx$$
$$\Rightarrow\; \int (5x^2-2)^{1/2}(2x)\,dx \;=\; \int u^{1/2}\frac{du}{5} \;=\; \frac{1}{5}\cdot\frac{1}{3/2}\cdot u^{3/2} \;+\; C \;=\; \boxed{\frac{2(5x^2-2)^{3/2}}{15} \;+\; C}$$

Solution to 3d:

$$u(x) \;= x^6 - 8 \;\Rightarrow\; \frac{du}{dx} \;=\; 6x^5 \;\;\Rightarrow\;\; du \;=\; 6x^5\,dx \;\;\Rightarrow\;\; dx \;=\; \frac{du}{6x^5}$$
$$\Rightarrow\; \int \frac{4x^5}{x^6-8}dx \;=\; \int \frac{4x^5}{u}\,\frac{du}{6x^5} \;=\; \frac{4}{6}\int \frac{1}{u}\,du \;=\; \frac{2}{3}\ln|u| \;+\; C \;=\; \boxed{\frac{2}{3}\ln(x^6-8) \;+\; C}$$

Solution to 3e:

We substitute $u = x - 2$. Then $du = dx$ and

$$x(x-2)^5 \;= (u+2)u^5 \;=\; u^6 + 2u^5$$
$$\Rightarrow\; \int x(x-2)^5\,dx \;=\; \int u^6 + 2u^5\,du \;=\; \boxed{u^7/7 \;+\; u^6/3 \;+\; C}$$

Solution to 3f:

We substitute $u = 5t$. Then $du = 5\,dt$, and

$$\int e^{5t}\,dt \;= \frac{1}{5}\int e^{5t}(5)\,dt \;=\; \frac{1}{5}\int e^u\,du \;=\; \frac{1}{5}\,e^u + C \;=\; \boxed{\frac{1}{5}\,e^{5t} \;+\; C}.$$

Exercise 4*: The marginal profit in thousands of dollars as a function of items sold is $P'(q) \;=\; 3q(q^2+2)^2$. The profit from selling 30 items was \$10,000.00. Find the equation for the total profit.*

Solution to** 4**:

We obtain the profit function $P(q)$ by doing an $\int$ ...dq on the marginal profit $P'(q) \;=\; 3q(q^2+2)^2$ (thousands of dollars):

$$u(p) \;= q^2 \;+\; 2 \;\;\Rightarrow\;\; \frac{du}{dq} \;=\; 2q \;\;\Rightarrow\;\; \frac{3du}{2} \;=\; (3q)\,dq$$
$$\Rightarrow\;\; P(q) \;=\; \int (q^2+2)^2(3q)\,dq \;=\; \int u^2\,\frac{3du}{2} \;=\; =\; \frac{3}{2}\cdot\frac{1}{3}\cdot u^3 \;=\; \frac{(q^2+2)^3}{2} + C$$

*We still must compute C. For that we shall finally use the fact that the profit from selling 30 items was \$10,000.00 which translates into $P(30) = 10$ and **NOT** $P(30) = 10,000$ because the profit function $P(q)$ measures profit not in* *dollars but in thousands of dollars:*

$$P(30) \;=10 \;\;\Rightarrow\;\; \frac{(30^2+2)^3}{2} + C \;=\; 10 \;\;\Rightarrow\;\; \frac{902^3}{2} + C \;=\; 10 \;\;\Rightarrow\;\; C \;\;=\; 10 \,-\, 366{,}935{,}404.00$$

$$=\; -\,366{,}935{,}394.00 \;\;\Rightarrow\;\; \boxed{P(q) \;=\; \frac{(q^2+2)^3}{2} -\, 366{,}935{,}394.00}$$