Math 220 - Calculus f. Business and Management - Worksheet 33

Solutions for Worksheet 33 - Integration by Parts

Exercise 1: *Look at each function below. Assume that you will use integration by parts to find the integral of those functions. Choose which part of each function will be u and which part will be dv. Don't forget to include dx with the part that will be dv. Note that some functions may require some change in their form before choosing u and dv.*

1a $(x+6)e^{2x}$ ***1b*** $\dfrac{2x-3}{e^{5x}}$ ***1c*** $5x^3\ln x$ ***1d*** $\dfrac{2\ln x}{x^5}$ ***1e*** $\log_3 x$ ***1f*** $x^4\ln\sqrt[3]{x}$

Exercise 2:

Using the functions for u and dv in problem 1, find du and v. Remember that dx will appear with the function for du.

Exercise 3:

Find the integral of each function in problem 1.

Solution to exercises 1, 2, 3:

***Solution to** a**:** $(x+6)e^{2x}$

$u = x+6 \rightsquigarrow du = dx;\ v' = e^{2x} \rightsquigarrow v = \int e^{2x}dx = (1/2)e^{2x}$. *Hence*

$$\int(x+6)e^{2x}dx = uv - \int vdu = \frac{x+6}{2}e^{2x} - \int\frac{1}{2}e^{2x}dx$$
$$= \frac{x+6}{2}e^{2x} - \frac{1}{4}e^{2x} + C = \boxed{(\frac{x+6}{2} - \frac{1}{4})e^{2x} + C}$$

***Solution to** b**:** $\dfrac{2x-3}{e^{5x}}$

We rewrite this as $(2x-3)e^{-5x}$. *Let us choose* $u = 2x-3 \rightsquigarrow du = 2dx$ *and* $v' = e^{-5x} \rightsquigarrow v = (1/-5)e^{-5x}$ *because* e^{-5x} *is easy to integrate and* $u' = 2$ *does not add any complexity:*

$$\int(2x-3)e^{-5x}dx = uv - \int vdu = \frac{2x-3}{-5}e^{-5x} - \int\frac{2}{(-5)}e^{-5x}dx$$
$$= \frac{3-2x}{5}e^{-5x} - \frac{2}{(-5)}\frac{1}{(-5)}e^{-5x} + C = \boxed{\frac{3-2x}{5}e^{-5x} - \frac{2}{25}e^{-5x} + C}$$

***Solution to** c**:** $5x^3\ln x$*:*

We choose $u = \ln(x) \rightsquigarrow du = \dfrac{dx}{x}$ *and we choose* $dv = 5x^3 \rightsquigarrow v = \dfrac{5}{4}x^4$. *So,*

$$\int 5x^3\ln x\,dx = uv - \int v\,du = \ln(x)\frac{5}{4}x^4 - \int\frac{5}{4}x^4\frac{dx}{x} = \ln(x)\frac{5}{4}x^4 - \frac{5}{4}\int x^3\,dx = \boxed{\ln(x)\frac{5}{4}x^4 - \frac{5}{4}\frac{x^4}{4} + C}$$

***Solution to** d**:** $\dfrac{2\ln x}{x^5}$*:*

We rewrite this as $(2x^{-5})\ln(x)$. $u = \ln x \ \rightsquigarrow\ du = dx/x$;
$v' = 2x^{-5} \ \rightsquigarrow\ v = 2\int x^{-5}dx = (2/-4)x^{-4} = (-1/2)x^{-4}$. *Hence*

$$\begin{aligned}\int (2x^{-5})\ln(x)dx \ &= uv \ - \int vdu \ = \ -\frac{x^{-4}}{2}\ \ln(x) \ - \int \frac{-x^{-4}}{2x}dx \\ &= \ -\frac{x^{-4}}{2}\ \ln(x) \ + \ \frac{1}{2}\int x^{-3}dx \ = \ -\frac{x^{-4}}{2}\ \ln(x) \ + \ \frac{1}{2}\frac{1}{(-2)}x^{-2} \ + \ C. \\ &= \boxed{-\frac{x^{-4}}{2}\ \ln(x) \ - \ \frac{x^{-2}}{4} \ + \ C}.\end{aligned}$$

Solution to** e**: $\log_3 x$:

You can think of the solution to this problem as the "rule" for integrating $\log_b(x)$.

We choose $u = \log_3(x) \rightsquigarrow du = \dfrac{dx}{x\ln(3)}$. *We choose* $dv = 1$ *(this is a common trick) so that* $v = x$. *Therefore,*

$$\begin{aligned}\int \log_3(x)\,dx \ &= uv - \int v\,du \ = \ \log_3(x) - \int x\ \frac{dx}{x\ln(3)} \\ &= \ \log_3(x) - \frac{1}{\ln(3)}\int dx \ = \ \boxed{\log_3(x) - \frac{1}{\ln(3)}x \ + \ C}\end{aligned}$$

Solution to** f**: $x^4 \ln \sqrt[3]{x}$

We rewrite this as $(x^4/3)\ln(x)$. $u = \ln x \ \rightsquigarrow\ du = dx/x;\ v' = x^4/3 \ \rightsquigarrow\ v = \int x^4dx/3 = (1/15)x^5$. *Hence*

$$\begin{aligned}\int \frac{x^4}{3}\ \ln(x)dx \ &= uv \ - \int vdu \ = \ \frac{x^5}{15}\ \ln(x) \ - \int \frac{x^5}{15x}dx \\ &= \frac{x^5}{15}\ \ln(x) \ - \ \frac{1}{15}\int x^4dx \ = \ \boxed{\frac{1}{15}\Big(x^5\ \ln(x) \ - \ \frac{x^5}{5}\Big) \ + \ C}\end{aligned}$$