Math 220 - Calculus f. Business and Management - Worksheet 38

Solutions for Worksheet 38 - Investments and Money Flow

Exercise 1*: Calculate these one-time investment problems (use calculators to get an approximate answer).*

1a) What is the value of a single investment of $\$1000$ *at an interest rate of* 5% *per year for a period of* 5 *years?*

1b) How much needs to be invested today at 2% *per year in order to have* $\$2000$ *after* 10 *years?*

Solution to $1a$ ***and*** $1b$***:***

Problem		
1a	PV = 1,000	FV = PV * e^(0.05*5)
	$1,000.00	$1,284.03
1b	FV = 2,000	PV = FV * e^(-0.02*10)
	$2,000.00	$1,637.46

Figure 1: Problem 1 computations

Exercise 2*: Find the future value for these investments.*

2a) How much will be in the bank if money is invested at a rate of $\$200$ *per year for* 5 *years at a rate of* 5%*?*

2b) What is the future value of an investment of $1000e^{.01t}$ *at a rate of* 3% *for* 10 *years?*

Solution to $2a$***:*** *Continuous money flow is* $\$200$ *per year;*

Accumulated amount of money flow is

$$\begin{aligned} P(5) &= e^{.05(5)} \int_0^5 200e^{-.05t}dt \;=\; 200\, e^{.25} \int_0^5 e^{-.05t}dt \\ &= 200\, e^{.25}(1/-.05)e^{-.05t}\Big|_0^5 \;=\; -4000\, e^{.25}\left(e^{-.25}-1\right) \\ &= 4000\left(e^{.25}-1\right) \end{aligned}$$

Solution to $2b$***:*** *The wording of the problem is not very clear about this but we interpret* $\$1,000e^{.01t}$ *as the continuous money flow (investment* <u>*rate*</u>*) at time* t*: Future value*

$$\begin{aligned} P(10) &= e^{.03(10)} \int_0^{10} (1000e^{.01t})e^{-.03t}dt \;=\; 1000\, e^{.3} \int_0^{10} e^{-.02t}dt \\ &= 1000\, e^{.3}(1/-.02)e^{-.02t}\Big|_0^{10} \;=\; -50,000\, e^{.3}\left(e^{-.2}-1\right) \;=\; 50000\, e^{.3}\left(1-e^{-.2}\right) \end{aligned}$$

Exercise 3*: Find the present value of both the investments in question 2.*

Solution to 3: In both cases we obtain the present value from the future value by multiplying it with e^{-rT}.

Solution to $3a$***:*** $P(0) \;=\; e^{-.25}P(5) \;=\; \$4,000\left(1-e^{-.25}\right)$

Solution to $3b$***:*** $P(0) \;=\; e^{-.3}P(10) \;=\; \$50,000\left(1-e^{-.2}\right)$

Exercise 4: *Choose the correct formula to answer each of these questions.*

a) A retiree decides to buy an annuity that will pay $\$4000$ *per year for* 20 *years. The interest rate on the annuity is* 6%. *How much will it cost to purchase this annuity?*

b) A young couple puts $\$10,000$ *in the bank for their child's education. At* 4% *interest, how much will they have at the end of* 15 *years?*

c) Grandparents want to put some money in the bank for their grandchild's first car. How much money must they put in now in order to have $\$12,000$ *in* 8 *years? Interest rates are* 3%.

d) A student begins saving at a rate of $\$240e^{.02t}$ *per year. At an interest rate of* 5%, *how much will the student have after* 5 *years?*

Solution to $4a$***:*** *I have an issue with the wording of the problem: I am not a financial expert and do not understand the meaning of "The interest rate on the annuity is...". Considering the subject matter of the chapter on money flow I interpret this as "the risk-free interest rate on which the discount value of the annuity is based because I could invest at that rate rather than purchase the annuity": look at example 35.3 in the text (p.274) where you are asked to judge whether this is a good investment based on a* <u>*bank*</u> *interest rate of* 7% *at which you could alternatively invest your money. Under this assumption, here is the solution: Present value of the investment flow based on a bank interest rate is*

$$\begin{aligned} PV &= \int_0^{20} \left(4000e^{-.06t}\right)dt \;=\; 4000\,(1/-.06)e^{-.06t}\Big|_0^{20} \\ &= -\,66,666.67\left(e^{-.12}-1\right) \;=\; \$66,666.67\left(1-e^{1.2}\right) \\ &\approx \$66,666.67\cdot 0.699 \;\approx\; \$46,587.05 \end{aligned}$$

Solution to $4b$***:*** *Straightforward computation of the future value of a one-time investment:*

$$P(15) \;= e^{.04(15)}10000 \;=\; \$10,000\,e^{.6}$$

Solution to $4c$***:*** *Straightforward computation of the present value of a one-time investment:*

$$PV \;= e^{-.03(8)}12000 \;=\; \$12,000\,e^{-.24}$$

Solution to $4d$***:*** *Accumulated amount of money flow is*

$$\begin{aligned} P(5) &= e^{.05(5)}\int_0^5 \left(240\,e^{.02t}\right)e^{-.05t}dt \;=\; 240\,e^{.25}\int_0^5 e^{-.03t}dt \\ &= 240\,e^{.25}(1/-.03)\,e^{-.03t}\Big|_0^5 \;=\; -8,000\,e^{.25}\left(e^{-.15}-1\right) \;=\; 8,000\,e^{.25}\left(1-e^{-.15}\right) \end{aligned}$$