Math 220 - Calculus f. Business and Management - Worksheet 40

Solutions for Worksheet 40 - 3–Space

Exercise 1:

Find an equation for the line that passes through $(2,7)$ and $(4,3)$ in the form $px+qy+r=0$. Use the techniques of elimination demonstrated in Section 25 in your textbook. Verify that your answer is correct by checking to see that each point satisfies your equation.

Solution to** 1**: *We treat p,q,r as unknown variables in the equation $px+qy+r=0$ and plug in first $x=2, y=7$ to get one equation for p,q,r and we plug in $x=4, y=3$ to get a second one for p,q,r:*

$$px+qy+r=0 \rightsquigarrow \begin{cases} x=2, y=7 & p(2)+q(7)+r= \quad 0 \\ x=4, y=3 & p(4)+q(3)+r= \quad 0 \end{cases}$$
$$\rightsquigarrow \begin{cases} \textbf{\textit{a)}} & 2p+7q+r= \quad 0 \\ \textbf{\textit{b)}} & 4p+3q+r= \quad 0 \end{cases}$$

*Subtract **a) - b)**: $-2p+4q+0=0 \rightsquigarrow p=2q$*
*Plug that into **a)**: $4q+7q+r=0 \rightsquigarrow r=-11q$.*
Plug $p=2q$ and $r=-11q$ into "$px+qy+r=0$":

$$2qx+qy-11qr=0 \rightsquigarrow q\big(2x+y-11r\big)=0 \rightsquigarrow \boxed{2x+y-11r=0}.$$

You may ask what if $q=0$? The answer: This would mean $px+0\cdot y+r=$, i.e., the line would be parallel to the y-axix. But that's impossible because the two points $(2,7)$ and $(4,3)$ on the line have different x-values.

We check our solution first for $x=2, y=7$: $2x+y-11r \ = \ 4+7-11=0$, so that's OK.
And now for $x=4, y=3$: $2x+y-11r \ = \ 8+3-11=0$, so that's OK too.

Exercise 2: *Find the equation for the plane that passes through each set of points in the form $px+qy+rz+s=0$. Use the techniques of elimination demonstrated in Section 25 in your textbook. Verify that your answer is correct by checking to see that each point satisfies your equation.*

$2a)$ $\quad (3,0,0),(2,1,0),(0,4,1)$

$2b)$ $\quad (1,4,1),(1,7,3),(3,9,1)$

Solution to** $2a$**:

NONE
NONE
NONE

Solution to** $2b$**: *We treat p,q,r,s as unknown variables in the equation $px+qy+rz+s=0$ and plug in first $x=1, y=4, z=1$ to get equation a for p,q,r,s. Then we plug in $x=1, y=7, z=3$ to get equation b for p,q,r,s. Finally we plug in $x=3, y=9, z=1$ to get equation c for p,q,r,s:*

$$px+qy+rz+s=0 \rightsquigarrow \begin{cases} x=1, y=4, z=1 \rightsquigarrow \textbf{\textit{a)}} & p+4q+r+s=0 \\ x=1, y=7, z=3 \rightsquigarrow \textbf{\textit{b)}} & p+7q+3r+s=0 \\ x=3, y=9, z=1 \rightsquigarrow \textbf{\textit{c)}} & 3p+9q+r+s=0 \end{cases}$$

We'll make substitutions that will express everything in terms of r:
Solve a) for s: $\boxed{s = - p - 4q - r}$.

Plug that into b):

$$p + 7q + 3r - p - 4q - r = 0 \ \rightsquigarrow 3q + 2r = 0 \ \rightsquigarrow \ \boxed{q = -(2/3)r}$$

Plug that into c):

$$\begin{aligned} 3p + 9(-1)(2/3)r + r - p - 4q - r = 0 \ &\rightsquigarrow 2p - 6r - 4(-2/3)r = 0 \\ &\rightsquigarrow 2p - (10/3)r = 0 \ \rightsquigarrow \ \boxed{p = (5/3)r} \end{aligned}$$

Plug p *and* q *into* $s = -p - 4q - r$*:* $s = (-5/3)r + (8/3)r - r \ \rightsquigarrow \ \boxed{s = 0}$.

At this point the only surviving variable is r *and we can plug* p, q, s*, all expressed as functions of* r*, into* $px + qy + rz + s = 0$*:*

$$\begin{aligned} px + qy + rz + s \ &= \frac{5}{3}r \cdot x \ - \ \frac{2}{3}r \cdot y \ + \ rz \ = \ 0 \\ &\rightsquigarrow \boxed{\frac{5}{3}x \ - \ \frac{2}{3}y \ + \ z \ = \ 0}. \end{aligned}$$

We check our solution by plugging our three points into this formula.
First for $x = 1, y = 4, z = 1$*:* $(5/3) \ - \ (2/3)(4) \ + \ 1 \ = \ (5 - 8 + 3)/3 \ = \ 0$ *and that checks out.*
Second for $x = 1, y = 7, z = 3$*:* $(5/3) \ - (2/3)7 \ + \ 3 \ = \ (5 - 14 + 9)/3 \ = = 0$ *and that checks out.*
Finally for $x = 3, y = 9, z = 1$*:* $(5/3)3 \ - (2/3)9 \ + \ 1 \ = \ (5 - 6 + 1) \ = \ 0$ *and that checks out too.*