Math 220 - Calculus f. Business and Mgmt - Worksheet 9

Worksheet 9 - Piecewise Defined Functions and Continuity

Evaluating and Graphing Functions

Exercise 1: Compose these pairs of functions into a single function

$$\textbf{1a}: f(x) = \begin{cases} 2x + 3 & \text{if } x < 3, \\ x^2 & \text{if } x \geq 3. \end{cases}$$ *Evaluate f(1), f(3), f(5) and graph the function.*

$$\textbf{1b}: g(x) = \begin{cases} 4x & \text{if } x < 2, \\ x + 3 & \text{if } x \geq 2. \end{cases}$$ *Evaluate g(-3), g(2), g(4) and graph the function.*

$$\textbf{1c}: h(x) = \begin{cases} 3x + 1 & \text{if } x < 1, \\ x + 3 & \text{if } x > 1. \end{cases}$$ *Evaluate h(-3), h(2), h(4) and graph the function.*

Building piecewise functions

Exercise 2: Labor costs $10 per hour until people have worked 40 hours. After 40 hours labor costs $15 per hour. Develop a piecewise function for cost as a function of hours worked: $c = C(t)$. Evaluate this function to find the cost of 35 hours of labor, then 40 hours and 45 hours. It may be easier to do the evaluation first and consider how the calculation was done before trying to define the function. Graph the function.

Exercise 3: A product is sold for $7.00 each unless it is bought in bulk. If 100 items or more of the product are purchased, the cost is $6.50 each. Develop a function $r = R(q)$ for revenue r as a function of items sold q. Find the revenue from the sale of 50 items, 100 items and 200 items. Graph the function.

Points of non-continuity

Exercise 4: For the following functions examine what happens at points where you would divide by zero and/or (the last two functions) where the function definition changes: Is there a limit? If so, is it finite? If not, what about the one-sided limits?

$$\textbf{4a}: \frac{5}{6x - 4}, \quad \textbf{4b}: \frac{3x + 2}{x^2 - x - 6}, \quad \textbf{4c}: \frac{x^2 - 2x - 35}{x^2 - 5x - 14},$$

$$\textbf{4d}: f(x) = \begin{cases} 1 - 2x & \text{if } x \leq 1, \\ 1/(x - 2) & \text{if } 1 < x < 3, \\ x + 2 & \text{if } x \geq 3; \end{cases} \qquad \textbf{4e}: f(x) = \begin{cases} x^2 + 4 & \text{if } x < 2, \\ (x + 6)/(x - 1) & \text{if } 2 < x \leq 4, \\ x - 2/3 & \text{if } x > 4. \end{cases}$$